

## Fracture toughness of honeycombs

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### Abstract

Crack growth initiation and subsequent resistance in regular and irregular hexagonal honeycomb structures made from ductile base materials are investigated in a numerical study. The elasto-plastic base material is characterized by a bilinear uniaxial stress-strain law describing the deformation behaviour prior to failure. The base material's fracture response is characterized through the fracture energy per unit area. Crack propagation is simulated by removing elements from a finite element model in a manner consistent with the energetics, and  $K$ -resistance curves are calculated under the assumption of small-scale yielding. Their dependence on the base material parameters and geometric imperfections of the structure are investigated.

### 1 Introduction

One of the failure mechanisms of metallic foam sandwich constructions is the initiation and growth of cracks in the foam core. In this context, a number of experimental studies have addressed recently the monotonic and fatigue failure of sandwich beams (eg.[1,2]). Previous studies of the fracture properties of cellular materials have been limited to the investigation of foams made from *brittle* base materials and the prediction of a critical stress intensity factor (eg. [3-7]).

The intention of this paper is to investigate the influence of the base material's elasto-*plastic* behaviour on the fracture properties of the cellular material. We consider regular and irregular hexagonal honeycomb structures as two-dimensional models of a metallic foam and calculate numerically  $K$ -resistance curves for the 2D foams under the assumption of small-scale yielding.

### 2 The model

Consider a macroscopic semi-infinite crack in a regular or irregular honeycomb (Fig. 1). The building elements of the hexagonal strut-network are slender beams (thickness  $t \ll$  average length  $l$ ) made from a ductile base material which, prior to failure, is described by the bilinear stress-strain relation

$$\begin{aligned} \epsilon &= \sigma/E & \sigma &< \sigma_y \\ \epsilon &= \sigma_y/E + (\sigma - \sigma_y)/H & \sigma &> \sigma_y \end{aligned} \quad (1)$$

in terms of the Young's modulus  $E$ , yield strength  $\sigma_y$  and hardening modulus  $H$ .

The relation (1) holds for stresses below the failure strength  $\sigma_f$ , which marks the beginning of the fracture process in the base material. This process is not modelled in detail; rather, we assume that it is sufficiently characterized by the fracture energy per cross-sectional area of the beam,  $\Gamma_0$ , which is regarded as a material parameter.

Under conditions of small-scale yielding, the displacements on a boundary remote from the crack tip are determined by the asymptotic mode I  $K$ -field and the value of the stress intensity factor  $K$ . Crack propagation is simulated by incrementally increasing the amplitude  $K$  of the boundary displacements until the failure strength  $\sigma_f$  is reached in one of the beams. This beam is then disconnected from the structure at the fracture location in such a manner that the energy dissipated during this process is compatible with the prescribed fracture energy  $\Gamma_0$  per unit cross-sectional area of the beam. The commercial finite element code ABAQUS is used to calculate stresses and strains in the structure due to the displacement boundary conditions dictated by the applied  $K$ , and the disconnection of beams upon fracture is performed by removing elements from the finite element mesh with a built-in routine.

A dimensionless stress intensity factor is introduced as

$$\tilde{K} = K/K^* \quad , \quad K^* = \sqrt{\pi l} \sigma_y (t/l)^2 \quad , \quad (2)$$

where  $K^*$  is an estimate for the stress intensity factor that causes initial yielding in the structure. It can be derived with the assumptions that the local deformation is bending dominated and that the displacement field around the crack tip is that of a linear elastic solid whose stiffness scales with  $(t/l)^3$  (cf. [3]). Dimensional analysis reveals that the dimensionless crack resistance  $\tilde{K}_R$  depends on the following set of dimensionless parameters:

$$\rho = t/l \quad , \quad \sigma_y/E \quad , \quad \sigma_f/\sigma_y \quad , \quad \gamma = H/E \quad , \quad \tilde{\Gamma} = \Gamma_0 E / (\sigma_y^2 l) \quad , \quad (3)$$

and, additionally, on statistical parameters describing the irregularity of the structure.

### 3 Results

Throughout the study, the material parameters were chosen as  $\sigma_y/E = 0.1\%$ ,  $\sigma_f/\sigma_y = 2.$ ,  $\gamma = 0.1$ ,  $\tilde{\Gamma} = 3.8$ , which is regarded as typical for the aluminium alloys from which metallic foams are made. As an example, Fig. 2 shows a deformed regular honeycomb after the failure of 15 beams. As a result of the structure's regularity, the crack follows a curved path oscillating about the plane of the macroscopic crack. Note that it is not possible to connect the initial and the current crack tip with a line without crossing intact beams. This can be regarded as crack-bridging and Fig. 2 shows that this bridging zone extends over the whole region of fractured beams up to the initial crack tip.

The corresponding crack resistance  $K_{IR}$  is plotted as a function of the crack extension  $\Delta a$  in Fig. 3 for two different 'relative densities'<sup>1</sup>  $t/l$ . The two curves differ only slightly; this

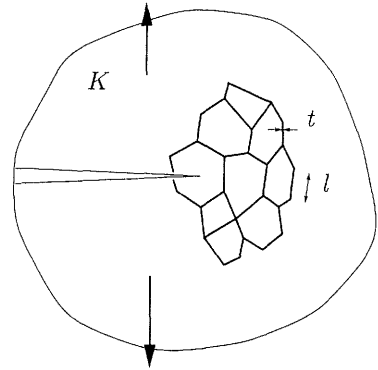


Figure 1: Small-scale yielding

<sup>1</sup>The true relative density is  $2/\sqrt{3}(t/l)$

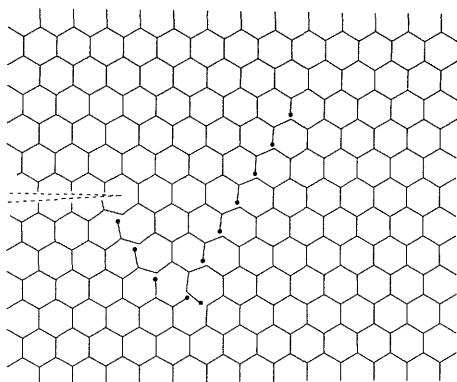


Figure 2: Crack path in a regular honeycomb

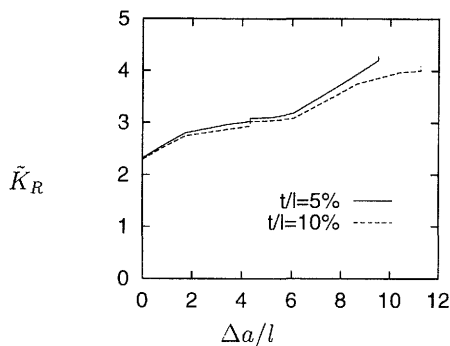
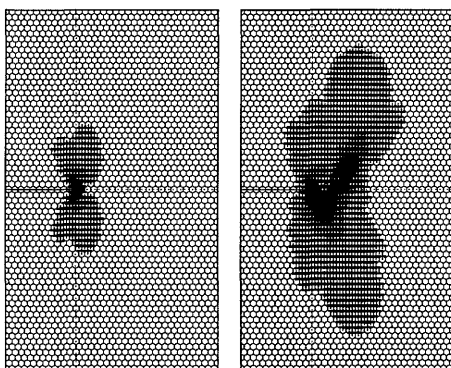


Figure 3: Crack growth resistance for two relative densities

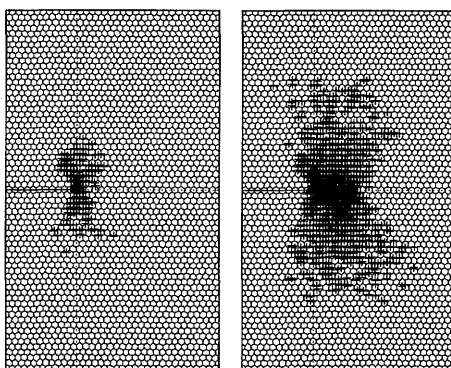
implies that the dependence of the crack resistance on the relative density is essentially captured in the non-dimensionalization (2), i.e. is quadratic to a good approximation. The size and shape of the plastic zone together with the magnitude of the plastic strains are illustrated in Fig. 4 for growth initiation and for  $\Delta a = 10l$ . The markers indicate the strut vertices where plastic yielding has occurred, their intensity corresponds to the three intervals of plastic strain  $\varepsilon_p < \varepsilon_y$ ,  $\varepsilon_y < \varepsilon_p < 5\varepsilon_y$  and  $\varepsilon_p > 5\varepsilon_y$ . For the regular structure appreciable plastic strains are confined to a narrow region around the crack. The shape of the plastic zone resembles that of a solid material in plain strain; the reason for this effect is that the yield strength of a regular hexagonal honeycomb under *biaxial* hydrostatic stress (which is present ahead of the crack tip) is much larger than under a deviatoric stress. Next, consider the case of an irregular (but still hexagonal) honeycomb. Fig. 6 shows a close-up of the crack tip region for a structure which is generated by randomly perturbing the vertex positions of a regular hexagonal honeycomb, the amount of pertur-



At initiation

At  $\Delta a = 10l$

Figure 4: Plastic zones and magnitude of plastic strains (regular honeycomb).



At initiation

At  $\Delta a = 10l$

Figure 5: Plastic zones and magnitude of plastic strains (irregular honeycomb).

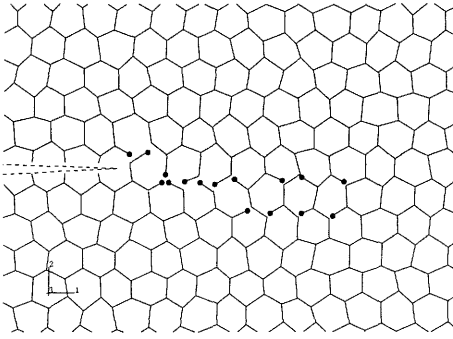


Figure 6: Crack path in an irregular honeycomb

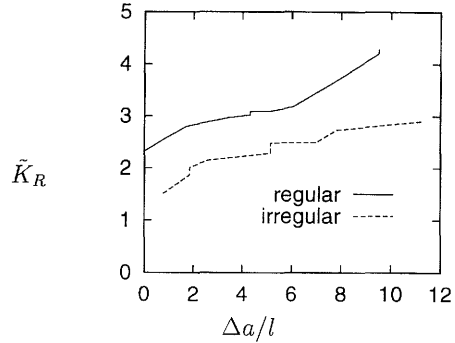


Figure 7: Crack growth resistance for regular/irregular honeycomb

bation being 15 % of the average beam length. Here, the oscillation of the crack path has vanished and the crack propagates more or less along the ligament. Also, the previously described bridging phenomenon is less significant. The crack resistance is considerably reduced, as compared to the regular honeycomb, and a steady state – or, at least, a very small slope – is attained for  $\Delta a = 8l$  (Fig. 7). Also, it is evident from Fig. 5 that the plastic zone shape is reminiscent of that for a fully dense solid in plane stress. This is due to the fact that the irregularities within the honeycomb depress the hydrostatic strength to a level comparable to the deviatoric strength (cf. [8]).

## 4 Conclusions

Crack growth resistance curves for two-dimensional cellular structures have been calculated for small-scale yielding conditions by employing a simple description of the elastoplastic base material and requiring that the local fracture process of individual cell walls be compatible with a prescribed fracture energy. The results show that the quadratic scaling of the toughness with the relative density, known already for elastic-brittle fracture [3], is retained in the range of material parameters considered here. For a regular hexagonal honeycomb the crack path oscillates around the ligament and a large bridging zone develops. Irregularity of the hexagonal structure reduces the toughness and a steady state is attained after the failure of only a few cell walls.

## References

- [1] Prasad, S., Carlsson, L. A., *Eng. Frac. Mech.*, 47(6), 813 (1994)
- [2] Burman, M., Zenkert, D., *Int. J. Fatigue*, 19(7), 551 (1997)
- [3] Gibson, L. J., Ashby, M. F., *Cellular Solids*, Pergamon Press, (1998)
- [4] Huang, J. S., Gibson, L. J., *Acta metall. mater.*, 39(7), 1627 (1991)
- [5] Huang, J. S., Gibson, L. J., *Acta metall. mater.*, 39(7), 1617 (1991)
- [6] Choi, J. B., Lakes, R. S., *Int. J. Fracture*, 80, 73 (1993)
- [7] Chen, J. Y., Huang, J. S., *J. Mech. Phys. Sol.*, 46(5), 789 (1998)
- [8] Chen, C., Lu, T. J., Fleck, N. A., to appear in *J. Mech. Phys. Solids* (1999)